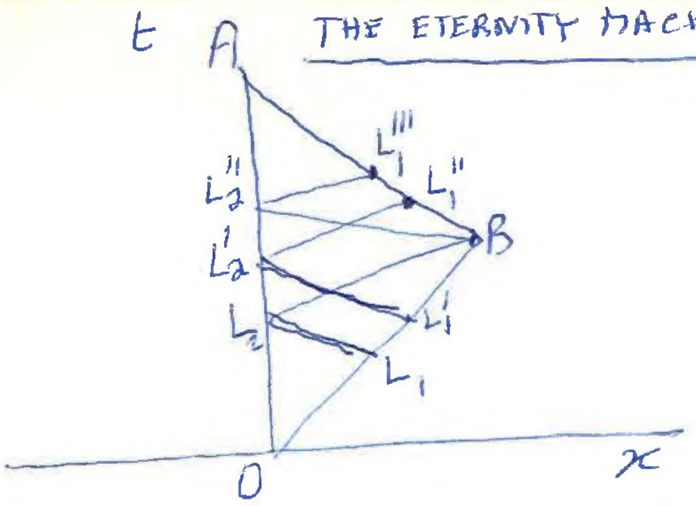
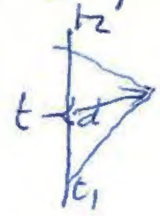
THE ETERNITY MACHINE ①




1 moves with velocity V along OB – returns along BA




$$t = \tfrac{1}{2}(t_2 + t_1)$$
$$d = \tfrac{1}{2}c(t_2 - t_1)$$

Eq OB is $t = \dfrac{x}{v}$ $\qquad OL_2' = T, \quad OA = 2T$

$B = (vT, T)$, $O = (0,0)$, $A = (0, 2T)$

$$OB = \sqrt{T^2 - \frac{v^2T^2}{c^2}} = T\sqrt{1 - v^2/c^2}$$

$$OB + BA = 2T\sqrt{1 - v^2/c^2}$$

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Eqn of $L_2B$ is $(t - T) = \frac{1}{c}(x - vT)$

So $L_2 = (0, T(1 - v/c))$

Eq. of $L_2L_1$ is $(t - T(1 - v/c)) = -\frac{1}{c}(x)$

$\therefore L_1 =$ soln of $\left.\begin{array}{l} t = x/v \\ t = T(1 - v/c) - \frac{x}{c} \end{array}\right\} \begin{array}{l} x = T\left(\frac{c-V}{c+V}\right)v \\ t = T\frac{c-V}{c+V} \end{array}$

$$t(L_2) = \tfrac{1}{2}\left(T + T\frac{c-V}{c+V}\right) = T\cdot\frac{c}{c+V}$$

$$o\,t(L_2) = \sqrt{1 - v^2/c^2}\cdot T\cdot\frac{c}{c+V} = \sqrt{\frac{c-V}{c+V}}\cdot T = \frac{1}{\sqrt{1-v^2/c^2}}T(1 - v/c)$$

and slope of $L_2, t(L_2) = \dfrac{\left(T\cdot\frac{c}{c+V} - T(1 - V/c)\right)}{\frac{1}{2}\left(vT + T\left(\frac{c-V}{c+V}\right)v\right)} = v/c^2$ ✓

and $d(L_2) = \frac{1}{2}c(OB - OL_1) = \frac{1}{2}c\,T\left(\sqrt{1-v^2/c^2} - \sqrt{1-v^2/c^2}\left(\frac{c-V}{c+V}\right)\right)$

$$= \frac{2}{2}cT\sqrt{1-v^2/c^2}\cdot\frac{V}{c+V}$$

$$= \cancel{T(1-v/c)\cdot vc\sqrt{1-v^2/c^2}\cdot\frac{1}{(1-v/c)^2}}$$

$$= T\,v\sqrt{1-v^2/c^2}\cdot\frac{c}{c+V}$$

Now write: $L_2 L_2' = \alpha$

so $O L_2' = T(1-v/c) + \alpha$.

$\alpha$ ranges from $0$ to $2vT/c$

Eq. of $L_1' L_2'$ is $\left(t - T(1-v/c) - \alpha\right) = -\frac{1}{c}(x)$

Eq. of $L_2' L_1''$ is $\left(t - T(1-v/c) - \alpha\right) = \frac{1}{c} x$

$L_1' L_2'$ intersects $OB$

where $\left.\begin{array}{l}\left(t - T(1-v/c) - \alpha\right) = -1/c \cdot x \\ t = x/v\end{array}\right\}$

so $t - T(1-v/c) - \alpha = -\frac{1}{c} vt$

or $t = \dfrac{T(1-v/c) + \alpha}{1+v/c}$

and so $L_2' L_1''$ intersects $AB$ where

and $\left.\begin{array}{l}\left(t - T(1-v/c) - \alpha\right) = \frac{1}{e} x \\ t - 2T \qquad = -1/v\, x\end{array}\right\}$

or $t = T(1-v/c) + \alpha \mp (t-2T)$

$\alpha\alpha t = T(1-v/c) + \alpha + 2T$

or $t =$

$t = T(1-v/c) + \alpha + \frac{1}{c}\left(-v(t-2T)\right)$

or $t\left(1 + \frac{v}{c}\right) = T(1+v/c) + \alpha$

or $t = \dfrac{T(1+v/c) + \alpha}{1+v/c}$

$$\therefore \frac{t_1+t_2}{2} = \frac{T+d}{1+v/c}$$

and $t(d) = \sqrt{1-v^2/c^2} \cdot \dfrac{T+d}{1+v/c}$

when $d=0$ $\quad t(d) = \sqrt{\dfrac{c-v}{c+v}} \cdot T$

$$= \frac{1}{\sqrt{1-v^2/c^2}} \cdot T(1-v/c)$$

slope of $t(d)$ line is $\sqrt{\dfrac{c-v}{c+v}} = \sqrt{\dfrac{1-v/c}{1+v/c}}$

So time lapse for ①

is
- up to $T(1-v/c)$ : $\dfrac{1}{\sqrt{1-v^2/c^2}} \cdot T(1-v/c)$
- from $T(1-v/c)$ to $T(1+v/c)$ : $2Tv/c \sqrt{\dfrac{c-v}{c+v}}$
- from $T(1+v/c)$ to $2T$ : $\dfrac{1}{\sqrt{1-v^2/c^2}} T(1-v/c)$

$$\text{Sum} = 2T\sqrt{\frac{c-v}{c+v}}\,(1+v/c)$$

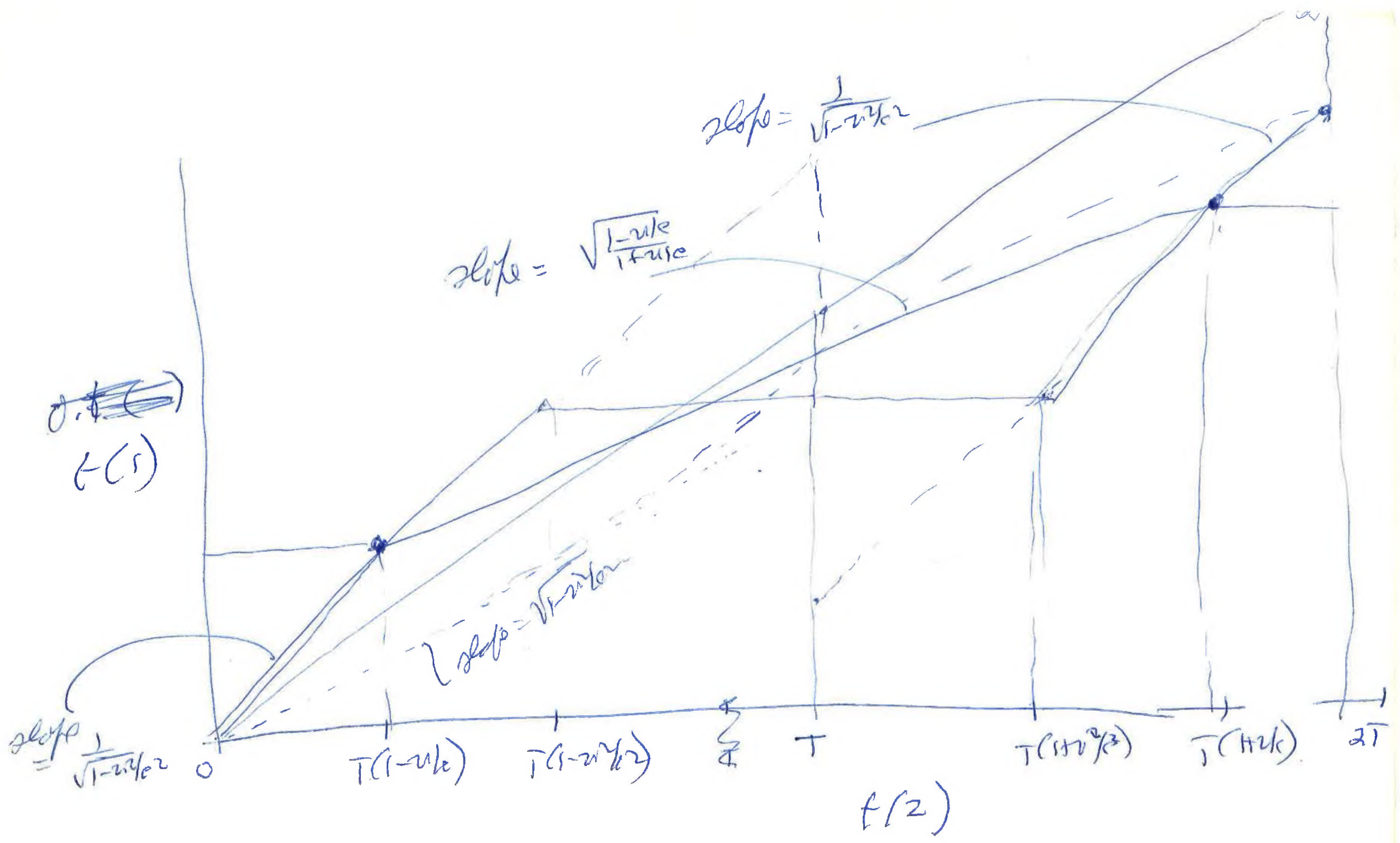
$$= \frac{2T}{c}\sqrt{c^2-v^2} = 2T\sqrt{1-v^2/c^2}$$

time for ② up to $T$ is $\dfrac{1}{\sqrt{1-v^2/c^2}} \cdot T(1-v/c) + T\sqrt{1-v^2/c^2}$

obtained by putting $d = v/c$ $\quad = 2T\sqrt{1-v^2/c^2}$

$\Rightarrow t(T) = \sqrt{1-v^2/c^2} \cdot T$

4



slope = $\frac{1}{\sqrt{1-v^2/c^2}}$
slope = $\sqrt{\frac{1-v/c}{1+v/c}}$
slope = $\sqrt{1-v^2/c^2}$
slope = $\frac{1}{\sqrt{1-v^2/c^2}}$
$t(1)$
0
$T(1-v/c)$
$T(1-v^2/c^2)$
$T$
$T(1+v^2/c^2)$
$T(1+v/c)$
$2T$
$t(2)$

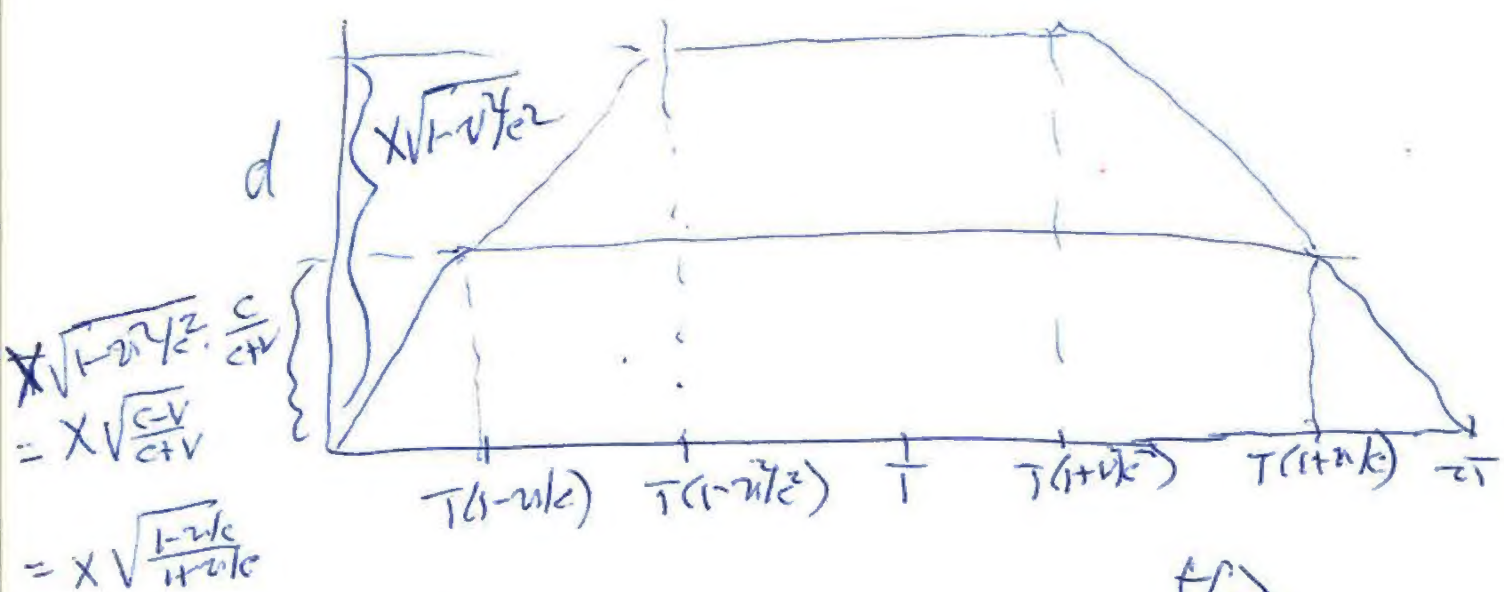
Notice that

$$\tfrac{1}{2}c(t_2 - t_1) = \frac{T v}{1 + v/c}$$

So $d(\alpha) = \sqrt{1-v^2/c^2} \cdot \frac{\cancel{2}T\,v\cancel{/c}}{1+v/c}\ \frac{T v}{1+v/c}$

independent of $\alpha$.

$$= T \cdot v\sqrt{1-v^2/c^2} \cdot \frac{c}{c+v}$$

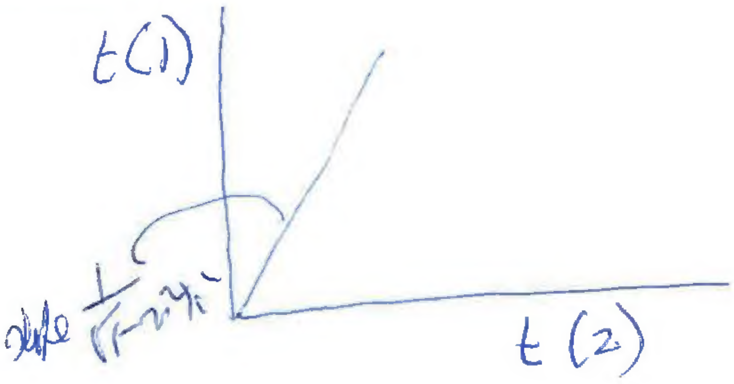



where $X = vT$

slope of $(d, t/2)$ curve is $\frac{X}{T}\frac{\sqrt{c^2-v^2}}{c+v} \cdot \frac{1}{1-v/c} = v \cdot \frac{c}{\sqrt{c^2-v^2}}$

$$= \frac{v}{\sqrt{1-v^2/c^2}}$$

Composite clock → $\varepsilon$-parameter or $m$-value 




(1) change $t(1)$ from proper time to 2's coord. time

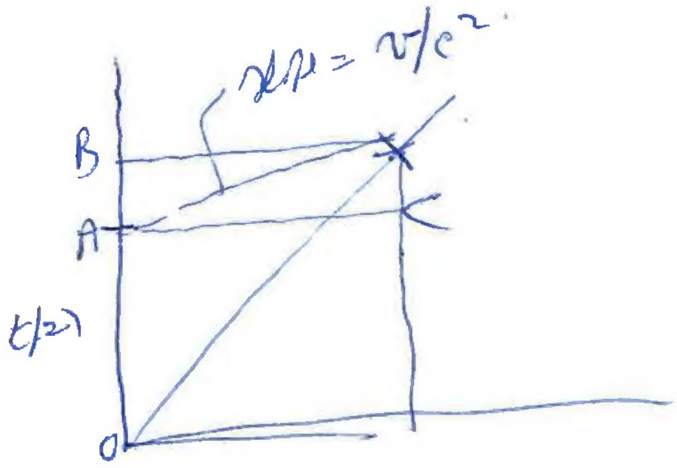
then slope $\rightarrow \frac{1}{\sqrt{1-v^2/c^2}} \times$ old slope

$= \frac{1}{1-v^2/c^2}$.

$s' =$ slope with proper time for (2)

$= \sqrt{1-v^2/c^2} \cdot s$

or $s = \frac{s'}{\sqrt{1-v^2/c^2}}$




slope of $t(1)$ v $t(2)$ curve in 2's coord. time

$= \frac{OB}{OA} = \frac{OA + AB}{OA} = 1 + \frac{AB}{OA}$

slope $m$ of line of simultaneity $= \frac{AB}{AC} = \frac{OB - OA}{OB \times v}$

$= \frac{1}{v}\left(1 - \frac{OA}{OB}\right)$

$= \frac{1}{v}\left(1 - (1 - v^2/c^2)\right)$

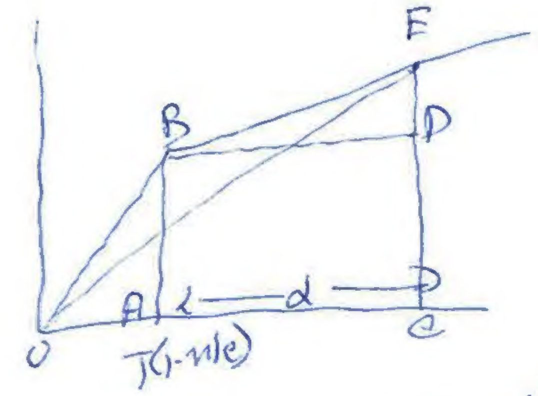
$= v/c^2$

So $m = \frac{1}{v}\left(1 - \frac{1}{s}\right)$

$= \frac{1}{v}\left(1 - \frac{\sqrt{1-v^2/c^2}}{s'}\right)$

along composite dead portion of $f(s)-f(1)$ curve 

$$S' = \frac{Ee}{de}$$




$$= \frac{ED + De}{OC} = \frac{\frac{1}{\sqrt{1-v^2/c^2}} \cdot T(1-v/e) + \sqrt{\frac{1-v/e}{1+v/e}}\left(t - T(1-v/e)\right)}{t}$$

$$= \sqrt{\frac{1-v/e}{1+v/e}} + \frac{T}{t}\left(\sqrt{\frac{1-v/e}{1+v/e}} - \sqrt{\frac{1-v/e}{1+v/e}}\,(1-v/e)\right)$$

$$= \sqrt{\frac{1-v/e}{1+v/e}} + \frac{T}{t}\cdot\frac{v}{c}\sqrt{\frac{1-v/e}{1+v/e}}$$

$$= \sqrt{\frac{1-v/e}{1+v/e}}\left[1 + \frac{T}{t}\cdot\frac{v}{e}\right]$$

and $m = \frac{1}{v}\left(1 - \sqrt{1-v^2/e^2}\,\sqrt{\frac{1+v/e}{1-v/e}}\left(1+\frac{T}{t}v/e\right)^{-1}\right)$

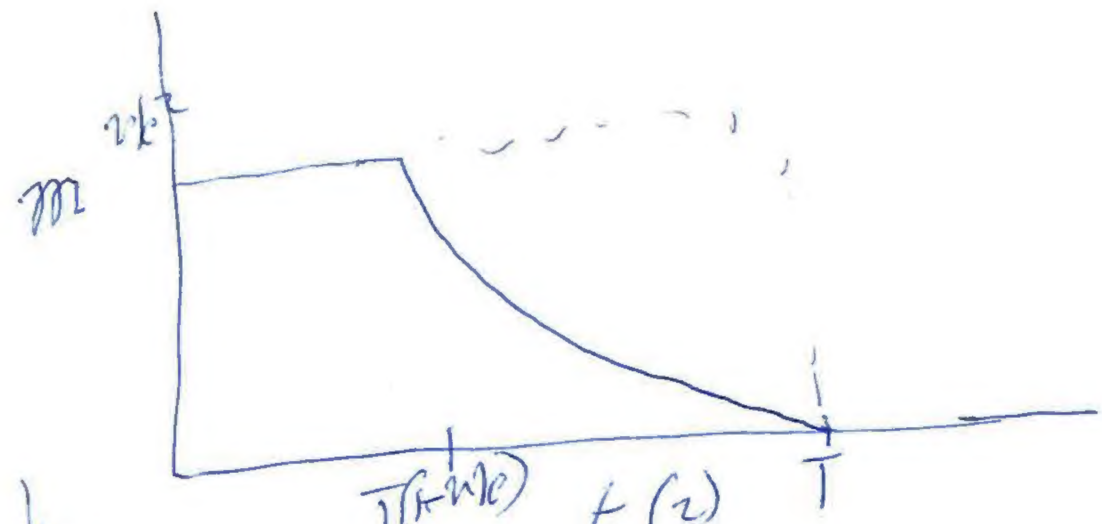
$$= \frac{1}{v}\left(1 - \frac{1+v/c}{1+\frac{T}{t}\cdot v/e}\right) \quad //$$

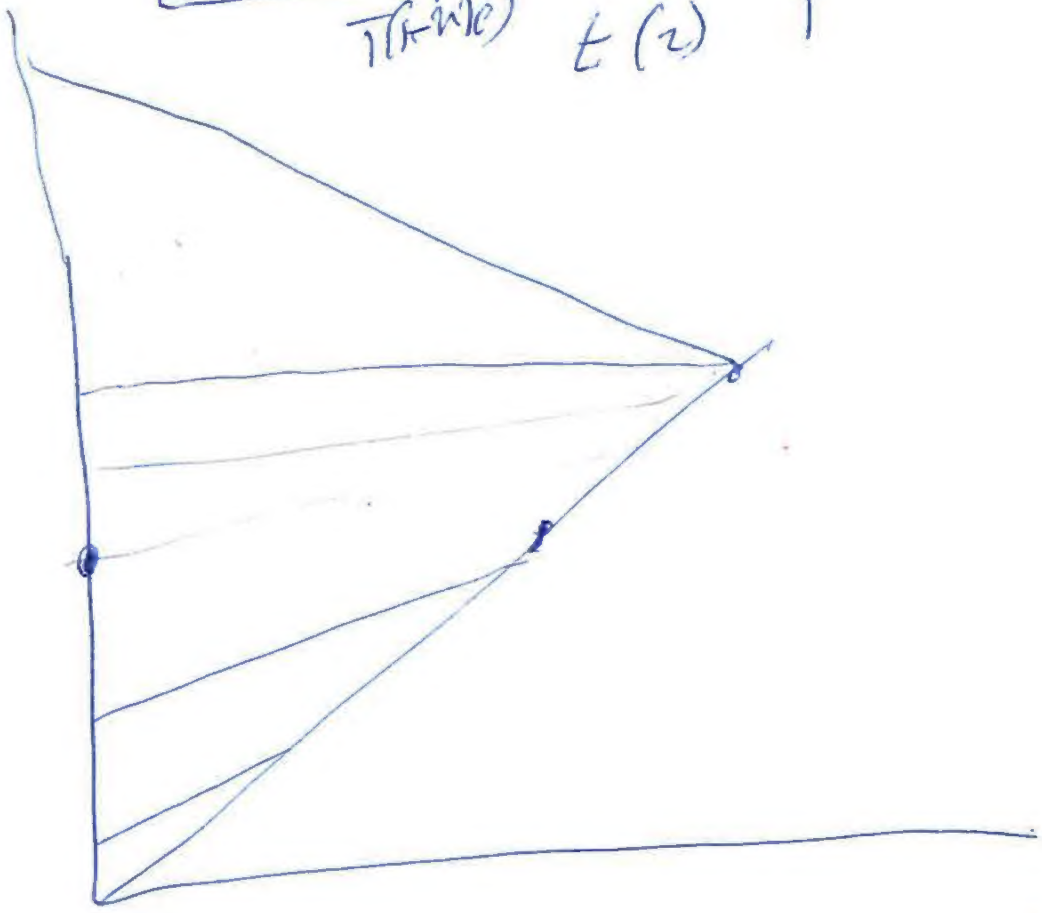
when $t = T$, $m = 0$

when $t = T(1-v/e)$, $m = \frac{1}{v}\left[1 - \frac{1+v/e}{1+\frac{v/e}{1-v/e}}\right]$

$$= \frac{1}{v}\left[1 - \left(1 - v^2/e^2\right)\right] = v/e^2.$$

Now, the connection between $m$ and $\varepsilon$ is



<u>P.T.P.</u> velocity $\dfrac{d'(p)}{\tau'(p)}$ of (2) as seen from (1)

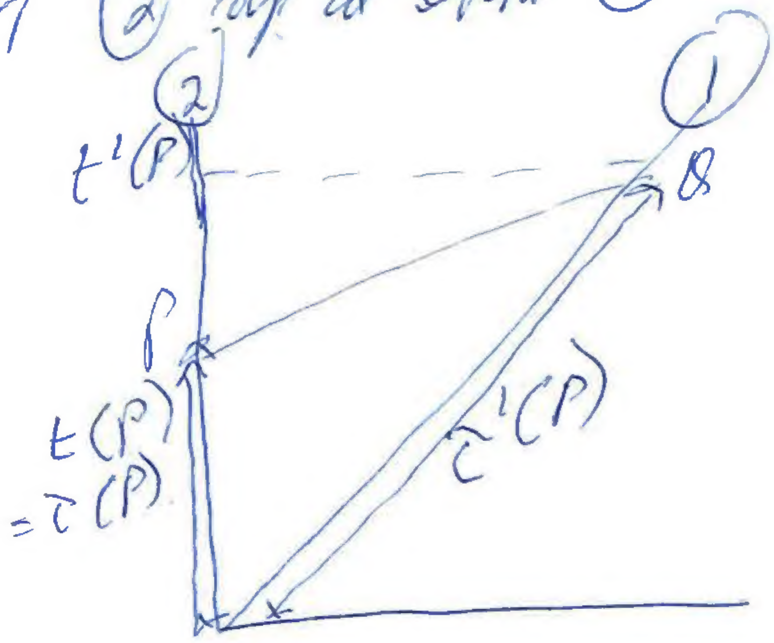
$\to 0$ since $d'(p)$ remains constant for $t(p) > T(1-v/c)$, $< T(1+v/c)$

# Improved notation

$t'(P)$ is time on world line of 1 (fast?) which is judged simultaneous with event P on worldline of 2, using 2's time coordinate

$\tau'(P)$ is proper time up to $t'(P)$ as measured by 1

$t(P) = \tau(P) =$ time (& proper time) measured by 2 up to event 2



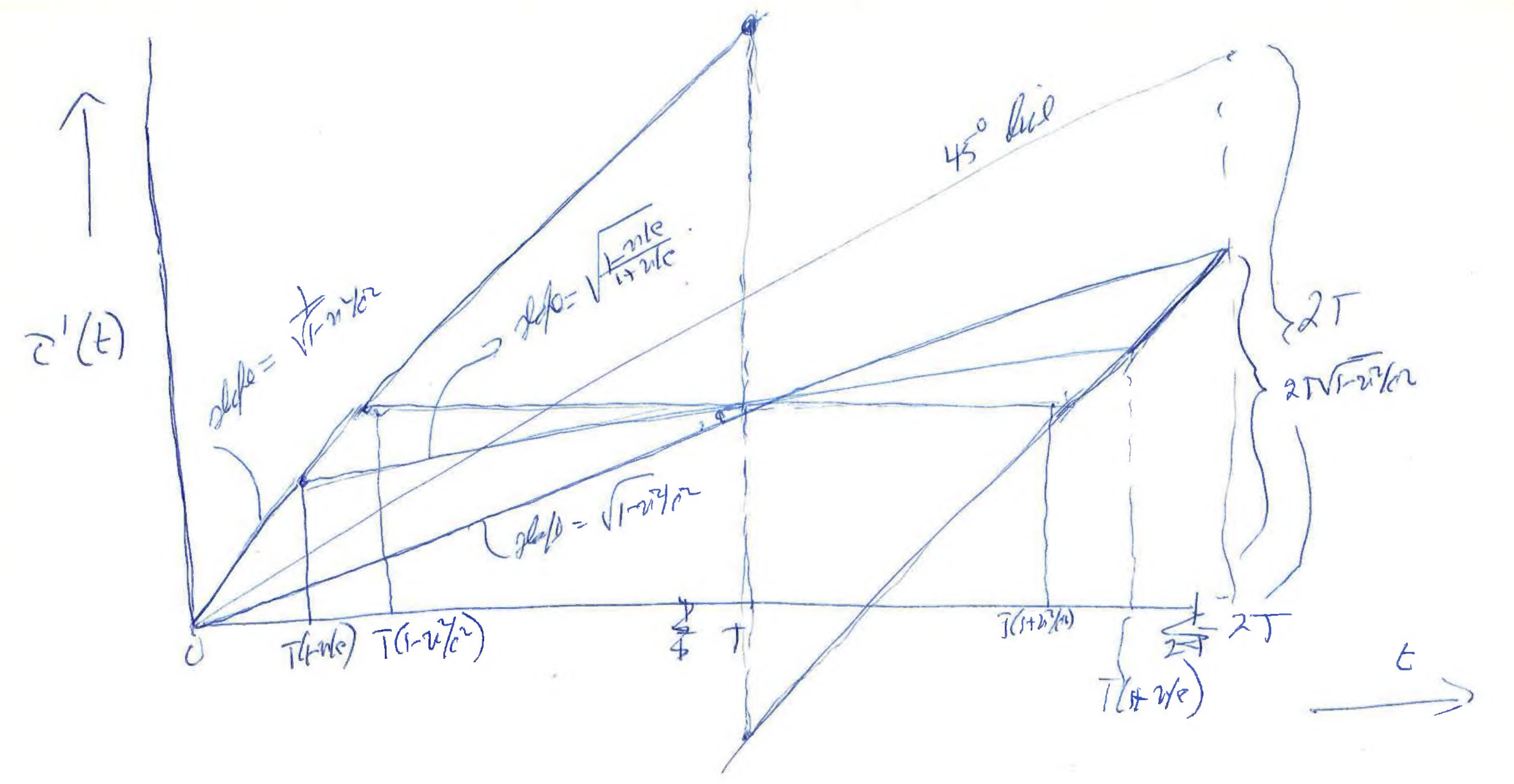

Don ran basic diagram plots $\tau'(P)$ versus $\tau(P)$ as P moves along world line of 2

$d'(P)$ is distance of 2 from 1 as seen from 1 at its proper time $\tau'(P)$

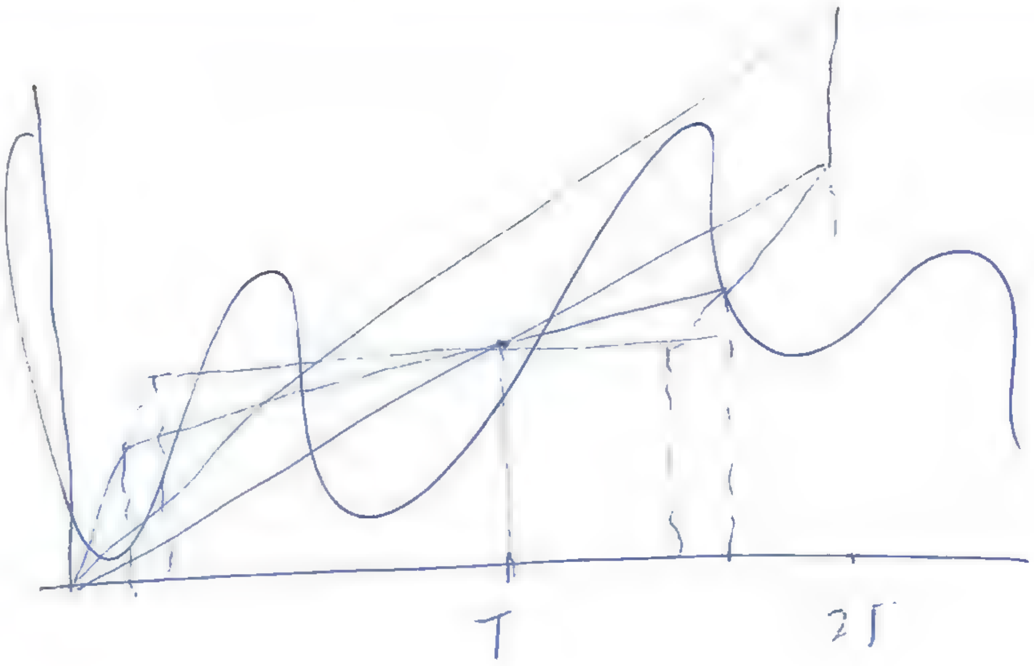
$s =$ slope of $t'(P) - t(P)$ curve

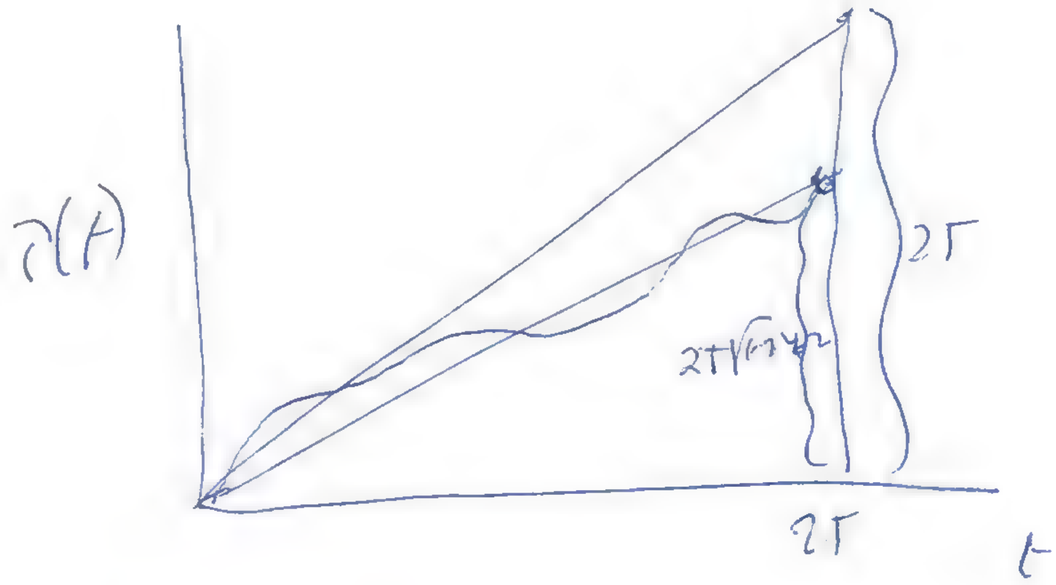
$s' =$ slope of $\tau'(P) - \tau(P)$ curve.

$m =$ slope of PQ in 2's reference frame.



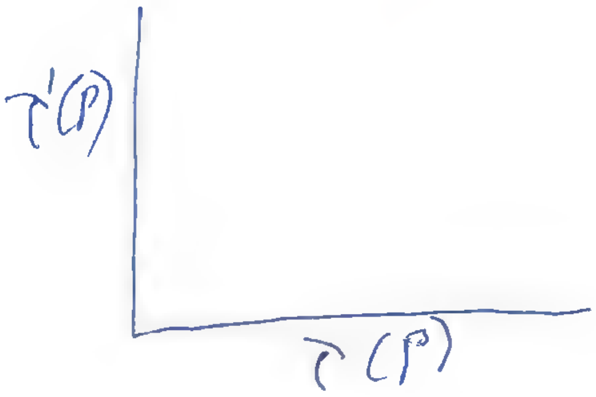
$\tau'(t)$
slope = $\frac{1}{\sqrt{1-v^2/c^2}}$
slope = $\sqrt{\frac{1-v/c}{1+v/c}}$
45° line
slope = $\sqrt{1-v^2/c^2}$
$2T$
$2T\sqrt{1-v^2/c^2}$
0
$T(1-v/c)$
$T(1-v^2/c^2)$
$T$
$T(1+v^2/c^2)$
$T(1+v/c)$
$2T$
$t$

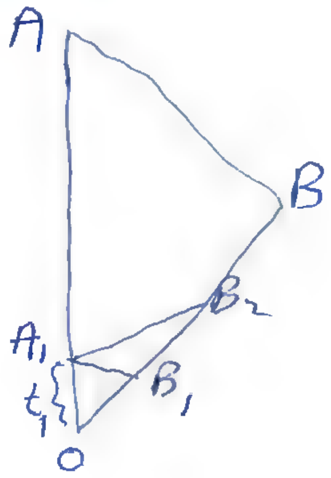



arbitrary reparametrization case:
cf. special case in previous diagram

we want now to determine upper and lower limits $\tau'_u(P)$ and $\tau'_e(P)$ of proper time for ① that is simultaneous with P on world line of ① by criterion of topological simultaneity




at point $A_1$, so $OA = t_1$, Eq. of $A_1B_1$ is

$$t - t_1 = -\frac{1}{c}x$$, intersects $OB$ world Eq. $t = x/v$

at the x-value $x/v - t_1 = -1/c\, x$. t-value is

or $x\left(\frac{1}{v}+\frac{1}{c}\right) = t_1$, so

$$\frac{1}{v}\left(\frac{t_1}{\frac{1}{v}+1/c}\right) \quad \text{and} \quad \tau'_e(t_1) = \sqrt{1-v^2/c^2}\cdot c\,\frac{t_1}{c+V}$$

$\boxed{\text{slope} = \sqrt{\frac{1-\beta}{1+\beta}}}$

Eq. of $A_1B_2$ is $t - t_1 = \frac{1}{c}x$, intersects $OB$ world Eq. $t = x/c$

at x-value, $x\left(\frac{1}{v}-\frac{1}{c}\right) = t_1$, so t-value

is $$\frac{1}{v}\left(\frac{t_1}{\frac{1}{v}-1/c}\right) \quad \text{and} \quad \tau'_u(t_1) = \sqrt{1-v^2/c^2}\cdot c\,\frac{t_1}{c-V}$$

$\boxed{\text{slope} = \sqrt{\frac{1+\beta}{1-\beta}}}$

these formulas apply until

$$\tau'_u(t_1) = \sqrt{1-v^2/c^2}\cdot T$$

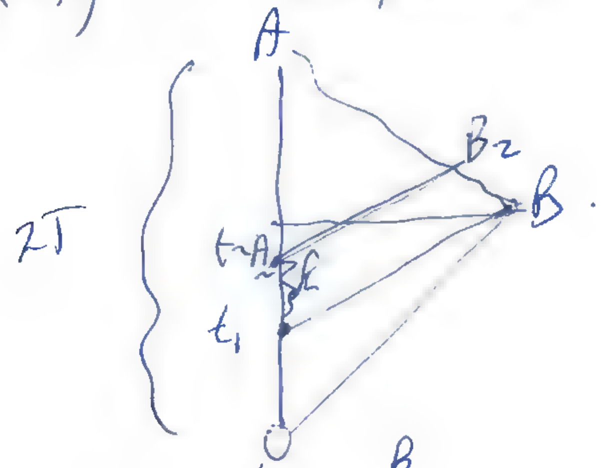
i.e. $\frac{ct_1}{c-V} = \cancel{c} \; T$ or $t_1 = T(1-v/c)$

$t_1 = T(1+v/c)$

and for $\tau'_e(t_1)$ until

for $t_1 > T(1-v/c)$

$\tau_u'(t_1)$ is computed as follows




write $t_2 = t_1 + h$.

Eq of $A_2 B_2$ is $t - t_2 = 1/c\, x$

Intersect $AB$ and Eq. $t - 2T = -\frac{1}{v} x$

at $x$-value $t_2 + \frac{x}{c} - 2T = -1/v\, x$.

i.e. $x\left(\frac{1}{c} + \frac{1}{v}\right) = 2T - t_2 = 2T - t_1 - h$

$= 2T - T(1 - v/c) - h$

$= T(1+v/c) - h$

So $t$-value for $B_2$ is

$t_2 + \frac{1}{c}\left[T(1+v/c) - h\right]\frac{cv}{c+v}$

$= T(1-v/c) + h + \frac{v}{c}\left[T(1+v/c) - h\right]\frac{1}{1+v/c}$

$= T - \cancel{Tv/c} + h + \cancel{\frac{v}{c} \cdot T \frac{1-v/c}{1+v/c}} - h\frac{v}{c} \cdot \frac{1}{1+v/c}$.

$= T + h\left(1 - \frac{v}{c} \cdot \frac{1}{1+v/c}\right) + \cancel{T\left\{\frac{v}{c} \cdot \frac{1-v/c}{1+v/c} - v/c\right\}}$

$\cancel{\{\ \} = \beta\frac{1-\beta}{1+\beta} - \beta = \beta\left(\frac{1-\beta}{1+\beta} - 1\right) = \beta\frac{(-2\beta)}{1+\beta} = -2\beta^2/1+\beta}$

and $= T + h\left(1 - \frac{\beta}{1+\beta}\right)$ $\quad \beta = v/c$

$= T + \frac{h}{1+\beta}$ , so when $h = 0$, $t(B_2) = T$

and when $h = 2T - T(1-\beta) = T(1+\beta)$

$T(B_2) = 2T$

as was required.

14

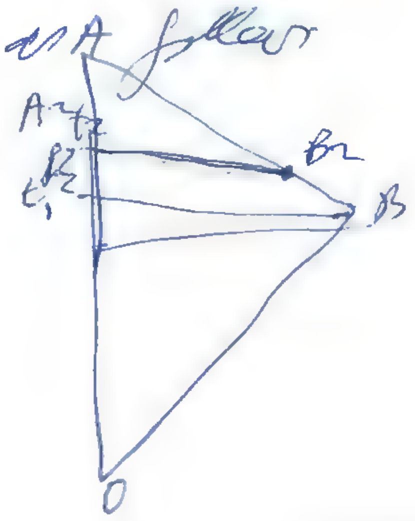
So, in interval of $t_1$ from $T(1-\beta)$ to $2T$

$$\tau_u(t_1) = \sqrt{1-\beta^2}\cdot T + \sqrt{1-\beta^2}\cdot \frac{h}{1+\beta}$$

slope of this line is $\frac{\sqrt{1-\beta^2}}{1+\beta} = \sqrt{\frac{1-\beta}{1+\beta}}$

While formula for $\tau_e(t_1)$ differs up to $t_1 = T(1+\beta)$ when it changes as follows

~~Now write~~




$t_1 = T(1+\beta)$

Again write $t_2 = t_1 + h$ where now

Eq. of $A_2B_2$ is $t - t_2 = -\frac{1}{c}x$

Inverse $AB$ unit eq. $t - 2T = -\frac{1}{u}x$

at $t$-value of $B_2$ (change $c \to -c$ and $+\beta$)

$$T + \frac{h}{1-\beta}$$

So when $h = 0$, $t(B_2) = T$

and when $h = 2T - T(1+\beta) = T(1-\beta)$, $t(B_2) = 2T$ as we require.

So, in interval of $t_1$ from $T(1+\beta)$ to $2T$

$$\tau_e(t_1) = \sqrt{1-\beta^2}\cdot T + \sqrt{1-\beta^2}\,\frac{h}{1-\beta}.$$

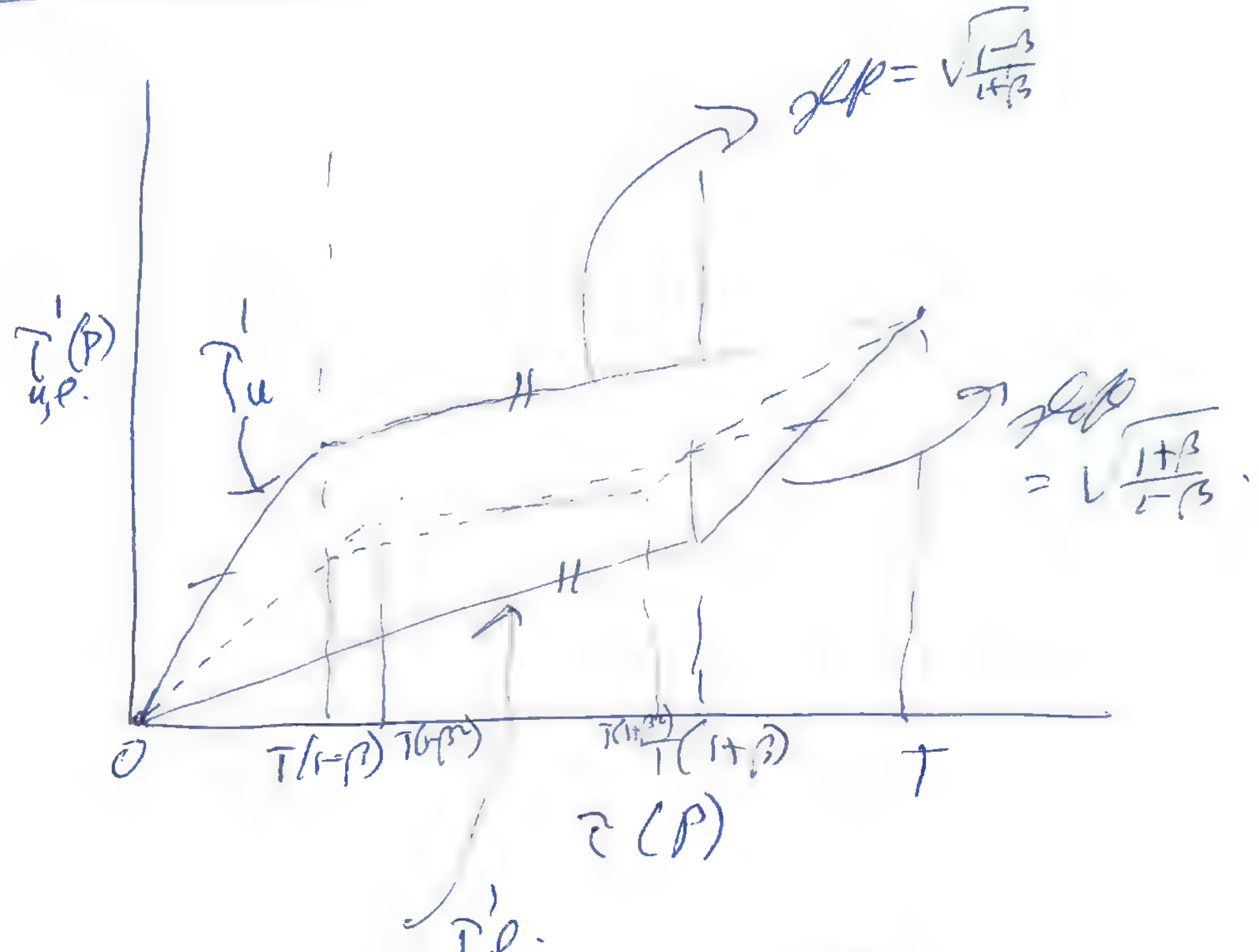
slope of this line is

$$\frac{\sqrt{1-\beta^2}}{1-\beta} = \sqrt{\frac{1+\beta}{1-\beta}}.$$

~~$\frac{\sqrt{1-\beta^2}}{1+\beta} = \sqrt{\frac{1-\beta}{1+\beta}}$~~

So, in summary graph of $\tau'_u(\beta)$ v. $\tau(\beta)$
and $\tau'_\ell(\beta)$ v. $\tau(\beta)$

looks like this:




... line shows average slope.

So up to $T(1-\beta)$ average slope

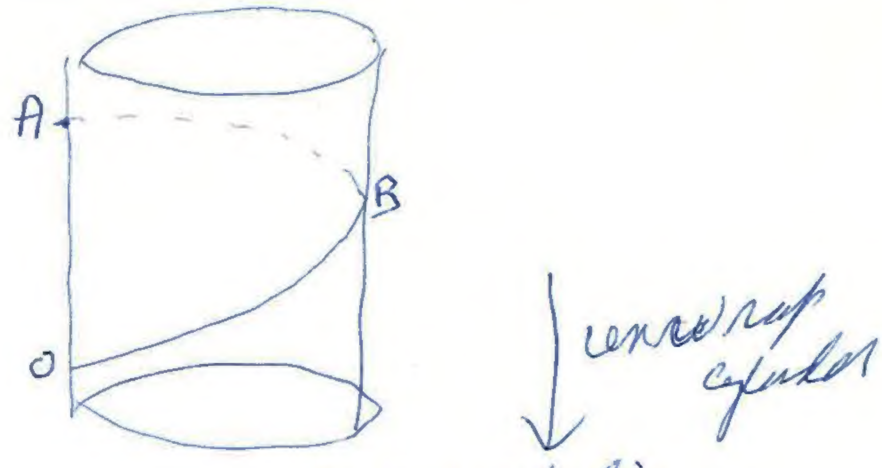
$$= \frac{1}{2}\left[\sqrt{\frac{1-\beta}{1+\beta}} + \sqrt{\frac{1+\beta}{1-\beta}}\right]$$

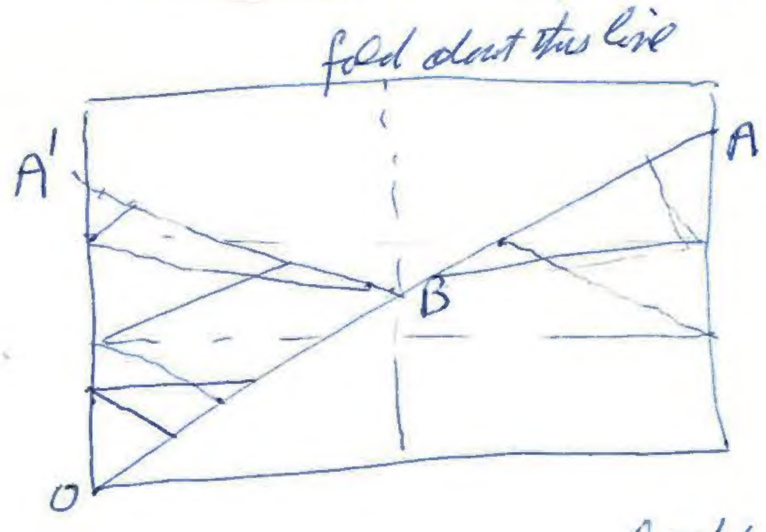
$$= \frac{1}{2\sqrt{1-\beta^2}}\left[1-\beta + (1+\beta)\right] = \frac{1}{\sqrt{1-\beta^2}}$$

This is also average slope in section $T(1+\beta)$ to $2T$

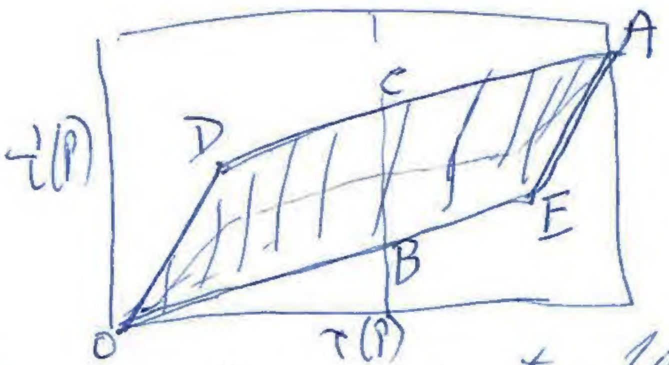
average slope in between $T(1-\beta)$ to $T(1+\beta)$ is just $\sqrt{\frac{1-\beta}{1+\beta}}$ as we found before.

## Simultaneity in a cylindrical Universe (16)

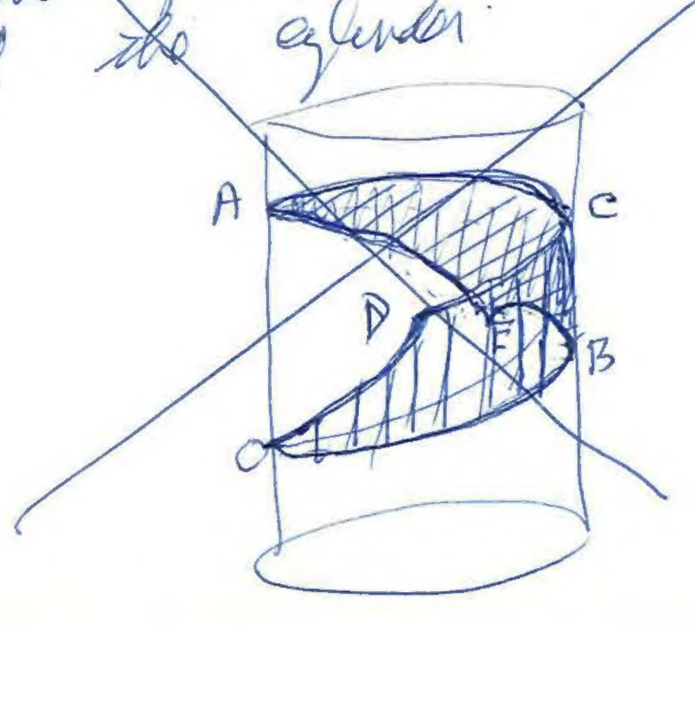


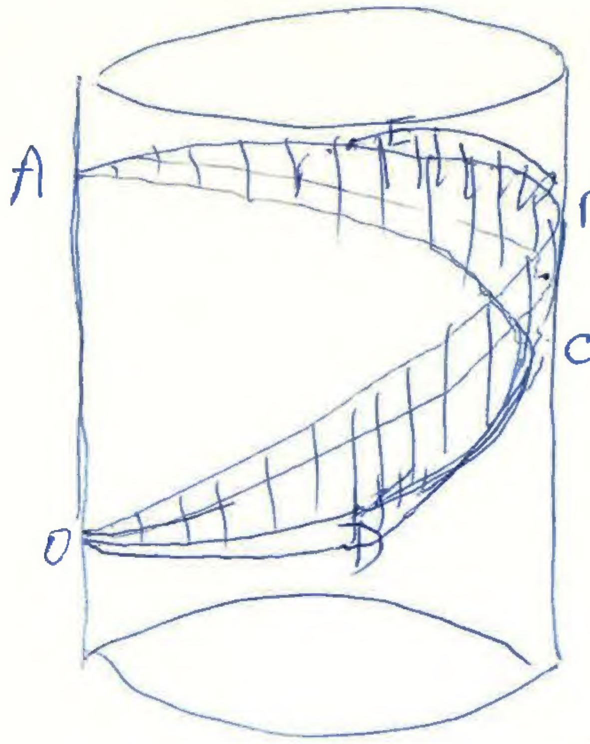




Exactly same diagram for latitude in simultaneity assignments as in the




The parallelogram is now to be wrapped round the cylinder





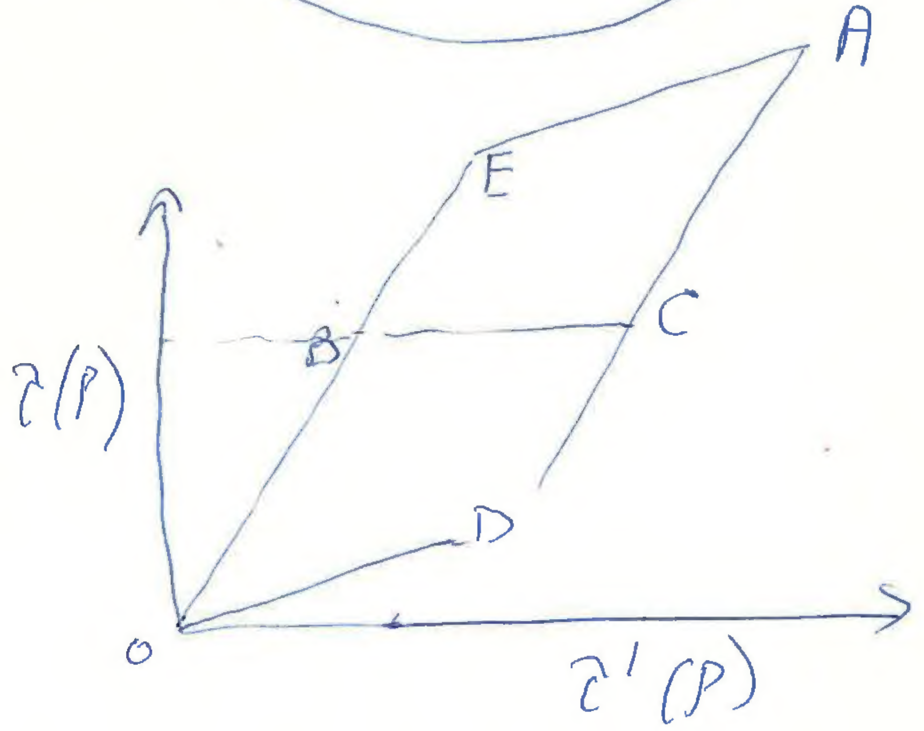
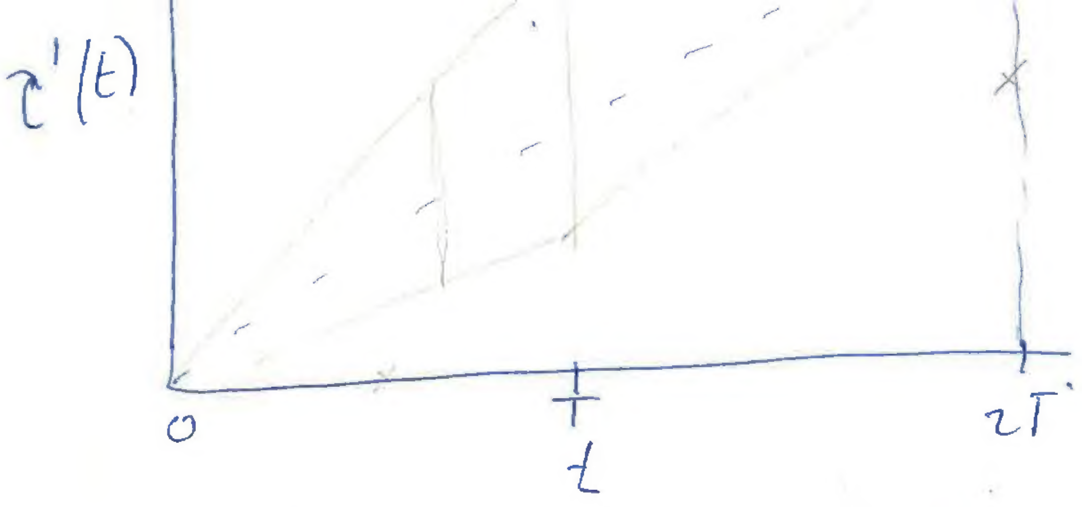
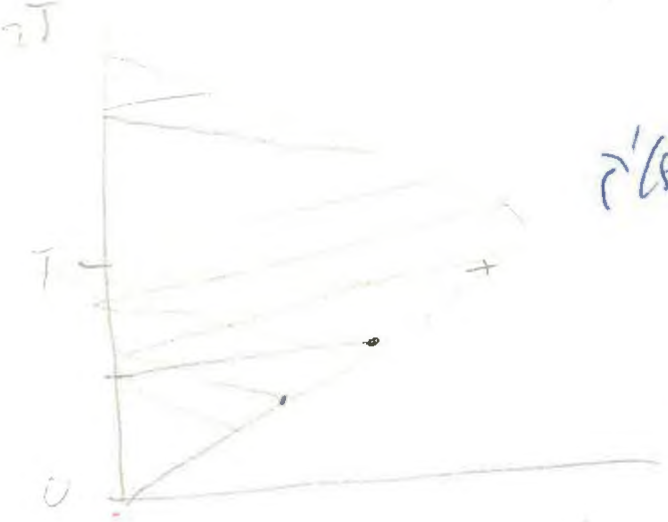
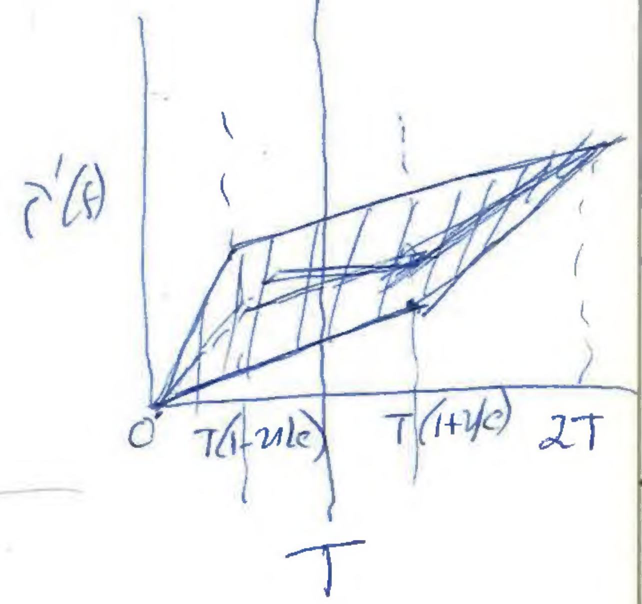
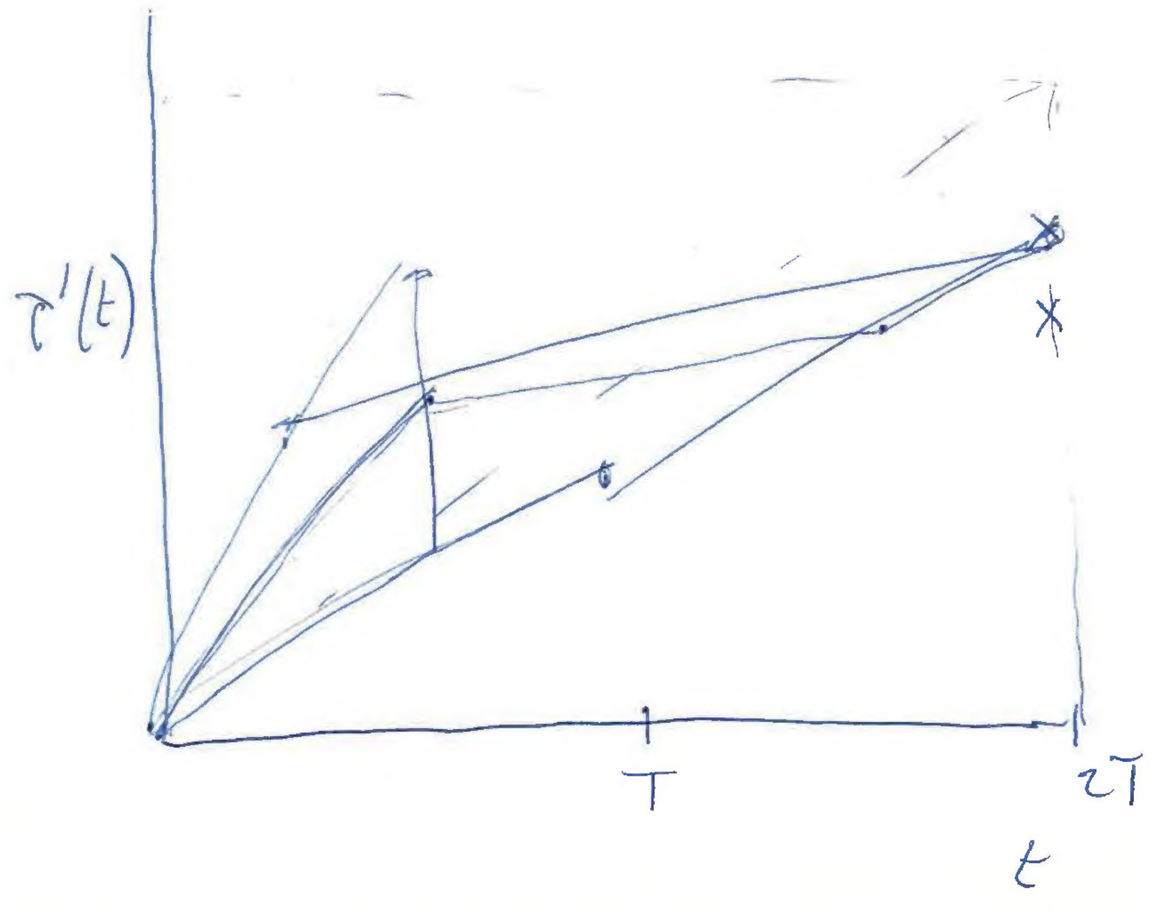




Diagram shows, as we follow path of moving 1, the latitude of $\tau(2)$ that could be allowed for simultaneity of a given point on 1's trajectory with time as measured by stationary 2.


τ'(t)
0
T
2T
t


2T
T
0


τ'(t)
0
T(1-u/c)
T(1+u/c)
2T
T


τ'(t)
T
2T
t

Does Q come before S along $\tau_\beta$ axis

value of $\tau_\beta$ for S is $T(1-v/c)\sqrt{\frac{1+v/c}{1-v/c}} = T\sqrt{1-v^2/c^2}$

value of $\tau_\beta$ for Q is $T(1+v/c)\sqrt{\frac{1-v/c}{1+v/c}} = T\sqrt{1-v^2/c^2}$.

So Fig 8 should look like this

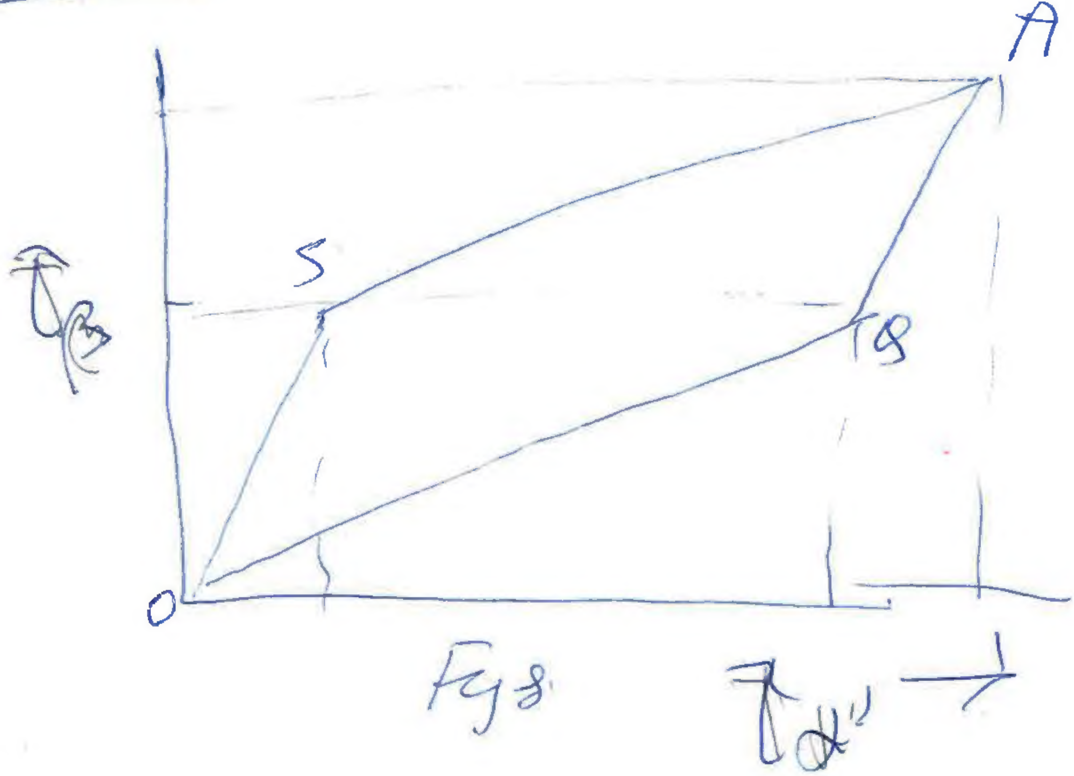



Fig 8.

SQ is $\parallel$ $\tau_\alpha$ axis

of length $2Tv/c$.

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In Fig 9 ordinate of F is $T(1-v^2/c^2)\frac{1}{2}\left(\sqrt{\frac{1+v/c}{1-v/c}} + \sqrt{\frac{1-v/c}{1+v/c}}\right)$

$= T(1-v^2/c^2)\frac{1}{2}\ \frac{(1+v/c)+(1-v/c)}{\sqrt{1-v^2/c^2}}$